Sample questions from the 2010 Olympiad

1. The lines L with equation $y = mx+b$ and M with equation $y = bx+m$ are perpendicular. If neither passes through the origin, find the product of their x-intercepts.

7. The sum of the infinite geometric series $a + ar + ar^2 + ar^3 + \ldots$ is 4, and the sum of the series whose terms are the squares of the terms of this series is 6. Find the sum of the infinite geometric series $a - ar + ar^2 - ar^3 + \ldots$

11. If $\cot C = k$ (a positive integer), express $\sin 2C$ as a fraction involving k.

15. For certain positive integers a, b, and c, $a^5 + b^2 + c^2 = 2010$. If b and c are prime, find $b + c$.

Solutions:

1. 1

7. 3/2

11. $2k/(k^2 + 1)$

15. $31 + 5 = 36$